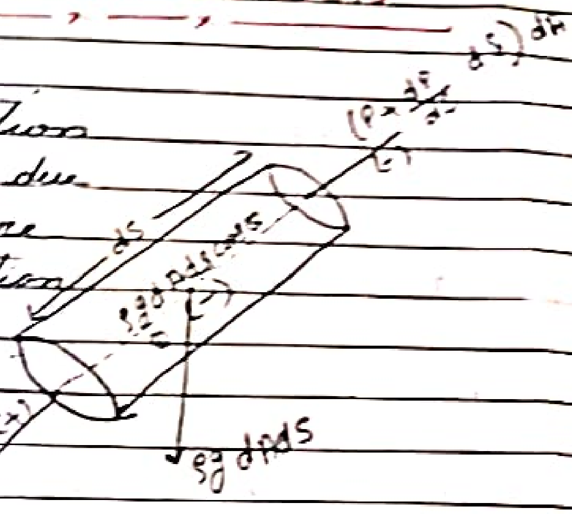
# EULER'S EQUATION OF MOTION

This equation of the motion in which the force due to gravity and pressure are taken in consideration. This is the derived by considering the motion of fluid element along a steam line.



(1) Pressure force in the direction of flow $= P$

(2) Pressure force opposite to direction of flow $=$

$$\left(P + \frac{dP}{ds}\, ds\right) dA$$

(3) weight of element $\rho g\, dA\, ds$

Resultant force $=$ mass of fluid element $\times$ accel. in the direction

$$PdA - \left(P + \frac{dP}{ds}\, ds\right) dA - \rho g\, dA\, ds \cos\theta = \rho \cdot dA\, ds \times a_s$$

$$\cancel{PdA} - \cancel{PdA} - \frac{dP}{ds}\, ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times a_s$$

$$-\frac{dP}{ds}\, ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times a_s \quad \text{--- (1)}$$

$a_s$ = Rate of change of velocity with respect to time.

$$a_s = \frac{dv}{dt}, \quad v\ \frac{ds}{dt}$$

Partial differential eqn.

$$\frac{dv}{ds} \cdot \frac{ds}{dt} + \frac{dv}{dt} \cdot \frac{dt}{dt}$$

$$\frac{dv}{ds} \cdot v + \frac{dv}{dt}$$

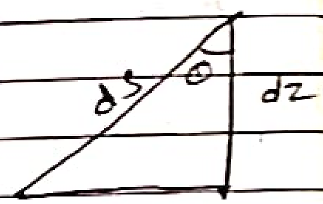
$$a_s = v\frac{dv}{ds} + \frac{dv}{dt}$$

Put all the value in eqn. (1)

$$-\frac{dp}{ds}\, ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times v \cdot \frac{dv}{ds}$$

Dividing by $\rho\, dA\, ds$

$$-\frac{\partial p}{\rho\, \partial s} - g\cos\theta = v\frac{dv}{ds}$$




$$\frac{\partial p}{\rho\, \partial s} + g\cos\theta + \frac{v\,dv}{ds} = 0$$

$$\frac{\partial p}{\rho\, ds} + g\frac{dz}{ds} + v\frac{dv}{ds} = 0$$

$$\frac{1}{ds}\left(\frac{\partial p}{\rho} + g\,dz + v\,dv\right) = 0$$

$$\boxed{\frac{dp}{\rho} + g\,dz + v\,dV = 0}$$

Euler's eqn

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